proposed that relatively poor performance with arithmetic problems in written-word format occurs because number-fact retrieval processes are less efficient with word compared to digit operands. Arithmetic problems are encountered more frequently in digit format (e.g., $2+6$, 4×5) than in written number-word format (two + six, four × five). Consequently, digit problems are more likely to activate a visual retrieval path than would a written-word problem (McNeil & Warrington 1994). Retrieval, given number words, presumably requires phonological recoding of problems so that the proximal retrieval cue is based on auditory-phonological codes. Retrieval with digits, therefore, would be more efficient because it is mediated both by well-established visual and phonological routes, whereas retrieval with number-word format would not provide a direct visual basis for retrieval.

The proposal that arithmetic-fact retrieval efficiency is lower with problems in word format than digit format is also supported by the well-replicated finding that non-retrieval strategies (i.e., procedural strategies such as counting or decomposition) are reported more often given word format (six + seven) than digit format ($6+7$) (Campbell & Epp 2005). Educated adults report procedural strategies for simple arithmetic up to 50% more with problems in written-word format than digit format (Campbell & Alberts, in press). Format-induced strategy shifts imply that different formats often recruit different neural processes for elementary arithmetic. Indeed, imaging research suggests that retrieval of arithmetic facts is associated with linguistic representations in the left angular gyrus, whereas procedural strategies requiring semantic quantity processing recruit bilateral components of the intraparietal sulcus (Dehaene et al. 2004). As direct retrieval and procedural strategies activate distinct brain regions (see also Dehaene et al. 2003), the effects of format on strategy choice for elementary arithmetic imply that calculation is not generally abstracted away from surface form.

Format-related strategy shifts demonstrate that calculation performance sometimes involves discrete, format-specific processes, but calculation also appears to be non-abstract in the second sense mentioned earlier; namely, that format-specific encoding processes or context can interact with calculation processes. One source of evidence for this comes, again, from research examining format effects on simple arithmetic: When procedural strategy trials are removed from analysis and only retrieval trials are analyzed, there remain substantial word-format costs relative to digit format, and word-format retrieval costs tend to increase with problem difficulty (Campbell et al. 2004; Campbell & Penner-Wilger 2006). This reinforces the conclusion that arithmetic retrieval processes are not abstracted away from surface format.

The non-abstractness of elementary arithmetic is demonstrated further by context-dependent activation of arithmetic facts. Bassok et al. (2008) found evidence for obligatory activation of addition facts ($4+2=6$) when problems were primed by word pairs semantically aligned with addition (e.g., tulips-daisies, which afford addition as a collection of flowers), but not when they were primed by pairs misaligned with addition (hens-radios, records-songs). The automaticity of arithmetic fact retrieval thereby depended on the analogical consistency of the semantic context activated by the prime and the specific arithmetic operation to be performed. This implies that the kinds and referents of problem operands are relevant to cognitive arithmetic, despite being irrelevant to arithmetic as a formal operation. Like the effects of surface form, semantic alignment phenomena demonstrate that cognitive arithmetic is not abstracted away from the conditions of problem encoding. Research on elementary arithmetic thereby aligns with the theoretical perspective represented in the target article, and points toward integrated, multimodal mechanisms in favor of abstract or amodal representations and processes (e.g., Barsalou 2008; Clark & Campbell 1991).

Numerical abstraction: It ain't broke

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Abstract: The dual-code proposal of number representation put forward by Cohen Kadosh & Walsh (CK&W) accounts for only a fraction of the many modes of numerical abstraction. Contrary to their proposal, robust data from human infants and nonhuman animals indicate that abstract numerical representations are psychologically primitive. Additionally, much of the behavioral and neural data cited to support CK&W's proposal is, in fact, neutral on the issue of numerical abstraction.

Cohen Kadosh & Walsh (CK&W) propose a new dual-code model of numerical representation that posits a psychological and neural distinction between fast, automatic notation-dependent representations and slower, intentional notation-independent representations. In their model, notation- and modality-specific (non-abstract) representations are psychologically more primary than abstract representations. We argue that this proposal is limited in its psychological and neurobiological perspective on numerical abstraction, and that the evidence they offer is either neutral on the issue of whether numbers are represented abstractly, or equally compatible with existing models of number representation.

A central limitation of CK&W's proposal is the coarse manner in which it surveys the theoretical landscape of numerical abstraction. At the psychological level, numerical abstraction can refer to notation independence, modality independence, or the representation of number independently of dimensions such as time, space, size, and color. For instance, the capacity to recognize that a group of three elephants is equal in number to a group of three umbrellas, but that both are fewer in number than a series of ten gunshots, is a feat of numerical abstraction. We know from scores of behavioral studies that human infants and nonhuman animals smoothly represent non-symbolic numerical values across modalities and dimensions (e.g., Cantlon & Brannon 2006; Church & Meck 1984; Hauser et al. 2002; Jordan & Brannon 2006; Jordan et al. 2005; 2008; Kobayashi et al. 2005; Nieder et al. 2006; Starkey et al. 1983; Wood & Spelke 2005). Importantly, infants and nonhuman animals exhibit these abstract numerical representations in the absence of symbolic language, and they do so spontaneously. Thus, numerical representations can be abstract in the absence of discrete symbolic representations or explicit task demands. CK&W's claim that "numerical representation is primarily non-abstract" (target article, Abstract) and that intentional processing is required to achieve notation- and modality-independent representations of numerical values is at odds with the demonstrated existence of this non-symbolic form of numerical abstraction.

Abstract non-symbolic numerical representations are important to any theory of numerical representation because they are hypothesized to provide the evolutionary and developmental foundation upon which symbolic numerical representations are psychologically constructed (e.g., Carey 2004; Gallistel & Gelman 2000). In other words, current developmental and evolutionary theories propose that numerical representations are abstract before they are symbolic. Therefore, CK&W's proposal needs to either (1) provide a theoretical account of the alleged developmental disappearance of automatic numerical abstraction in human children, or (2) make the case that preverbal infants and nonhuman animals spontaneously engage in intentional processing to represent numerical values across modalities and dimensions.

A second theoretical limitation of CK&W's proposal is that their neurobiological definition of numerical abstraction risks *reductio ad absurdum*. That is, the stipulation that numerical abstraction requires identical responses in identical neurons is potentially impossible to satisfy. Yet, even if it were possible to satisfy that criterion, it is not clear whether it is the appropriate criterion for establishing numerical abstraction. As the authors review, regions of the intraparietal sulcus (IPS) respond during numerical processing across notations, modalities, and dimensions. The mounting evidence that numerical representations across notations, modalities, and dimensions are "distributed but overlapping" in the IPS is neutral on the issue of whether the underlying representations are abstract. Instead, such evidence suggests that different numerical forms invoke both shared and separate neural processes. CK&W's conclusion that the neurobiological data weigh more heavily in favor of notation-dependent neural processes is therefore merely an assertion at this stage.

Other empirical evidence that CK&W cite in favor of their account does not do the theoretical work the authors are asking of it. The authors review both behavioral and neurobiological evidence purportedly revealing notation-specific interactions in numerical tasks. However, many of the notation-specific interactions they review hinge on generic differences in performance level. Specifically, if a single psychological process is involved in judging numerical values from two different numerical notations (e.g., numerical judgments of Arabic numerals and arrays of dots), yet the judgment is easier for one of the two notations (e.g., because the input mode is more rapid, reliable, or fluent), a notation-specific interaction may emerge simply because performance on the easier notation hit ceiling accuracy or floor speed. Such interactions, though cited by CK&W, do not invite the theoretical implications that CK&W draw. Instead, notation- or modality-specific interactions that arise under these circumstances reflect a quantitative difference in performance between notations or modalities. Note that this argument may also apply to neurobiological findings under circumstances in which floor or ceiling response levels are achieved. While bearing this issue in mind, we encourage CK&W to re-evaluate the relevance of the following studies to their argument for notation- and modality-dependent number representations: Dehaene and Akhavein (1995), Droit-Volet et al. (2008), Ganor-Stern and Tzelgov (2008), Hung et al. (2008), and Ito and Hatta (2003). These studies (and likely others) report interactions that do not necessarily support a notation- or modality-dependent account of numerical representation.

Importantly, any notation- or modality-dependent interaction that survives inspection for a generic performance effect likely can be accounted for by the two-system view of approximate and exact numerical representation proposed by Dehaene et al. (1999). In the pre-existing two-system proposal, notation-specific interactions may arise from an interplay between the exact and approximate numerical codes. Unfortunately, CK&W have not distinguished the empirical predictions of their dual-code view from the existing two-system view.

In short, although we applaud CK&W for highlighting some of the many remaining puzzles about the nature of numerical abstraction in the mind and brain, the solutions they offer do not adequately account for the data. Moreover, the open questions surrounding the cognitive and neural basis of numerical abstraction do not warrant a restructuring of the field of numerical cognition. Robust evidence demonstrates that with or without language, number is represented abstractly – independently of perceptual features, dimensions, modality, and notation. In fact, this is the very definition of "number."

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Numerical representations are neither abstract nor automatic

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Abstract: In this commentary, I support and augment Cohen Kadosh & Walsh's (CK&W's) argument that numerical representations are not abstract. I briefly review data that support the non-abstract nature of the representation of numbers between zero and one, and I discuss how a failure to test alternative hypotheses has led researchers to erroneously conclude that numerals automatically activate their semantic meaning.

There exists in the numerical cognition literature what I call the triple tautology: that numerical representations are (1) *automatically activated*, (2) *abstract*, and (3) *analogue*. Cohen Kadosh & Walsh (CK&W) present a convincing argument that numerical representations are not abstract. Although CK&W focus on the numerical representation of integers, strong evidence also exists for the non-abstract nature of the numerical representation of quantities between zero and one (Cohen et al. 2002). My colleagues and I have shown that, although most college students understand the correct ordinal relation of numbers expressed in a single numerical format (e.g., decimals), they do not understand the correct ordinal relation of numbers expressed in different formats (e.g., comparing decimals to relative frequencies). If the numerical representation of numbers between zero and one were abstract, the students should have been able to compare the semantic meaning of numbers expressed in different numerical formats once the numbers were converted into the abstract representation. Although researchers may discount this evidence for non-abstract representation of numbers as unique to those between zero and one, CK&W reveal that it is consistent with the evidence for the representation of integers.

The crux of CK&W's argument is that correlations should not be confused for causal mechanisms – no matter how intuitive the causal relations may appear. Below, I describe how the remaining two tautologies (automatic activation and analogue representation) also rely heavily on correlational evidence.

It can be argued that Moyer and Landaur (1967) started the modern study of numerical cognition with their discovery of the *numerical distance* effect. In short, the authors presented two integers side-by-side and asked participants to judge which integer was the larger of the two. The authors found that reaction time (RT) varied as a function of the numerical distance between the two presented integers. The robust nature of the finding, together with its appeal to our intuition about the importance of numerical distance, has made this finding one of the bedrocks of the numerical cognition literature. The numerical distance effect was the foundation of the first tautology of numerical cognition: the analogue nature of the representation.

The numerical distance effect is not only the foundation of the first tautology, but it is also a foundation of *automaticity*. A strong test of the automatic activation hypothesis is a simple task in which participants are to judge whether two numerical symbols are the same or different. In previous versions of this task, researchers dichotomized the stimuli into "close" and "far" groups by choosing numbers that are numerically "close" (e.g., 8 and 9) and numbers that are numerically "far" (e.g., 1 and 9). If semantic meaning is automatically activated, it will interfere with participants' same/different judgments and evidence for the numerical distance effect should be present in the RT data. Specifically, the time for participants to judge two numerically close numbers as different (i.e., the "close" group) should be longer than the time it takes them to judge two numerically distant numbers as different (i.e., the "far" group). This is